Unit10 Coordinate Geometry & Transformations

**7.1 Rigid motion in a plane**

* Figures in a plane can be reflected, rotated, or translated to produce a new figure. The new figure is called the **image**, and the original figure is called the **preimage**.
* The operation that maps, or moves, the preimage onto the image is called a **transfomation**.
* **Isometry** – a transformation that preserves lengths.
* When you name an image, take the corresponding point of the preimage and add a prime symbol.

 

* Three basic transformations



**7.2 Reflections**

* **Reflection –** a transformation which uses a line that acts like a mirror, with the image reflected in the line
* **Line of reflection –** the line which acts like a mirror in a reflection
* A figure in a plane has a **line of symmetry** if the figure can be mapped onto itself by a reflection in the line
* **Theorem 7.1 Reflection theorem -** A reflection is an isometry

|  |  |  |
| --- | --- | --- |
| **preimage A** | **line of reflection** | **image A’** |
| **(x, y)** | **x-axis y=0** | **(x , -y)** |
| **y-axis x=0** | **(-x , y)** |
| **y=x** | **(y , x)** |
| **x=#** | **(2# - x , y)** |
| **y=#** | **(x , 2# - y)** |

**7.3 Rotations**

* **Rotation** – a transformation in which a figure is turned about a fixed point
* **Center of rotation** – the fixed point of rotation
* **Angle of rotation** – angle formed by rays drawn from the center of rotation to a point and its image
* A figure in a plane has a **rotation symmetry** if the figure can be mapped onto itself by a clockwise rotation of 180˚ or less
* **Theorem 7.2 – rotation theorem** - A rotation is an isometry
* **Theorem 7.3** – if lines k and m intersect at point P, the reflection in k followed by a reflection in m is a rotation about point P. The angle of rotation is 2x˚, where x˚ is the measure of the acute or right angle formed by k and m

|  |  |  |
| --- | --- | --- |
| **preimage A** | **rotation clockwise** | **image A’** |
| **(x, y)** | **(angle of rotation, center of rotation)** |  |
| **(90⁰ , O) = 270⁰ counter clockwise** | **(y , -x)** |
| **(180⁰ , O)** | **(-x , -y)** |
| **(270⁰ , O) =90⁰ counter clockwise** | **(-y , x)** |
| **(360⁰ , O)** | **(x , y)** |

**7.4 Translations and vectors**

* **Translation** – a transformation that maps every two points P and Q in a plane to point P’ and Q’, so that the following properties are true:
	+ PP’=QQ’
	+ Segments PP’ and QQ’ are parallel or collinear
* **Vecto**r – a quantity that has both directions and magnitude (size)
* When a vector is drawn as PQ, the **initial point**, or starting point, of the vector is P and the **terminal point**, or ending point, of the vector is Q. PQ is read “vector PQ”
* **Coordinate notation** (x , y) →(x ± # , y ± #)
* The **component form** of a vector combines the horizontal and vertical components

 

* **Theorem 7.4 – translation theorem** - A translation is an isometry
* **Theorem 7.5** – if lines k and m are parallel, the reflection in k followed by a reflection in m is a translation. If P” is the image of P, then the following is true:
	+ PP” is perpendicular to k and m
	+ PP” = 2d, where d is the distance between k and m

**7.5 Glide Reflections and Compositions**

* **glide reflection** – a transformation in which every point P is mapped onto a point P” by the following steps
	+ A translations maps P onto P’
	+ A reflection in a line k parallel to the direction of the translation maps P’ onto P”
* **Composition of the transformations** – when two or more transformations are combined to produce a single transformation
* **Theorem 7.6** – The composition of two (or more) isometries is an isometry

**8.7 Dilations**

* a **dilation** with center C and scale factor K is a transformation that maps every point P in the plane to a point P’ so that the following properties are true
* if P is not the center point C, then the image point P’ lies on CP. The scale factor k is a positive number such that k=  ,and k≠1
* if P is the center point C, then P=P’
* The dilatation is a **reduction** if **0<k<1** and it is an **enlargement** if **k>1**

P(x, y) →P’(k\*x , k\*y)