UNIT 09 Solids

* **Polyhedron** (pl. polyhedra)– a solid that is bounded by polygons, called **faces**, that enclose a single region of space.
* An **edge** of a polyhedron is a line segment formed by the intersection of two faces.
* A **vertex** of a polyhedron is a point where three or more edges meet.
* **Regular** polyhedron – all of its faces are congruent regular polygons
* **Convex** polyhedron – if any two points on its surface can be connected by a segment that lies entirely inside or on the polyhedron
* **Cross section** – the intersection of a plane and a solid
* Types of solids
  + Polyhedra:
    - Prism
    - Pyramid
  + Not polyhedra:
    - Cylinder
    - Cone
    - sphere
* **Platonic solids** – (the 5 regular polyhedra)
  + **Tetrahedron** (4 triangle faces, 4 vertices, 6 edges)
  + **Cube** (6 square faces, 8 vertices, 12 edges)
  + **Regular octahedron** (8 triangle faces, 6 vertices, 12 edges)
  + **Regular dodecahedron** (12 pentagon faces, 20 vertices, 30 edges)
  + **Regular icosahedron** (20 triangle faces, 12 vertices, 30 edges)
* Theorem 12.1 – Euler’s Theorem - The number of faces (F), vertices (V), and edges (E) of a polyhedron are related by the formula F + V = E + 2
* **prism** – polyhedron with two congruent faces.
* **Bases** – the congruent faces of a prism
* **Lateral faces** – the other faces of a prism – parallelograms formed by connecting the corresponding vertices of the bases
* **Lateral edges** – edges form by connecting the corresponding vertices of the bases
* **Altitude or height of a prism** – the perpendicular distance between the bases
* **Right prism** – each lateral edge is perpendicular to both bases
* **Oblique prism** – has lateral edges that are not perpendicular to the bases
* **Slant height** – length of the oblique lateral edges
* **Net** – two-dimensional representation of all of the faces
* **Cylinder** – a solid with congruent circular bases that lie in parallel planes
* **Altitude or height of a cylinder** – the perpendicular distance between the bases
* **Right cylinder –** if the segment joining the centers of the bases is perpendicular to the bases
* **Lateral area of a cylinder** – the area of the curved surface – is equal to the product of the circumference and the height (2πrh)
* **Surface area of a cylinder** – is equal to the sum of the lateral area and the areas of the two bases
* Theorem 12.2 – Surface area of a right prism - The surface area S of a right prism can be found using the formula S=2B+Ph, where B is the area of the base, P is the perimeter of the base, and h is the height
* Theorem 12.3 – surface area of a right cylinder - The surface area S of a right cylinder is S=2B+Ch = 2πr2 +2πrh, where B is the area of a base, C is the circumference of the base, r is the radius of a base, and h is the height
* **pyramid** – polyhedron in which the base is a polygon and the lateral faces are triangles with a common vertex.
* **Lateral edges** – the intersection of two lateral faces
* **Base edges** – the intersection of the base and a lateral face
* **Altitude or height of a pyramid** – the perpendicular distance between the base and the vertex
* **Regular pyramid** – has a regular polygon for a base and its height meets the base at its center.
* **Slant height** of a regular pyramid – is the altitude of any lateral face
* **cone** – has a circular base and a vertex that is not in the same plane as the base
* **Altitude or height of a cone** – the perpendicular distance between the vertex and the base
* **Right cone –** the height meets the base at its center and the slant height is the distance between the vertex and a point on the base edge
* **Lateral area of a cone** – consists of all the segments that connect the vertex with points on the base edge (πrl where l is the slant height)
* Theorem 12.4 – Surface area of a regular pyramid - The surface area S of a regular pyramid can be found using the formula S=B + ½ Pl, where B is the area of the base, P is the perimeter of the base, and l is the slant height
* Theorem 12.5– surface area of a right cone - The surface area S of a right cone is S= πr2 +πrl, where r is the radius of the base, and l is the slant height
* **volume of a solid** – the number of cubic units contained in its interior. Volume is measured in cubic units (u3)
* **Postulate 27 – volume of a cube** – the volume of a cube is the cube of the length of its side, V=s3
* **Postulate 28 – volume congruence postulate** – if two polyhedra are congruent, then they have the same volume.
* **Postulate 29 – volume addition postulate** – the volume of a solid is the sum of the volumes of all its non-overlapping parts
* Theorem 12.6 – Cavalieri’s Principle – if two solids have the same height and the same cross-sectional areas at every level, then they have the same volume
* Theorem 12.7 – volume of a prism – the volume V of a prism is V=Bh, where B is the area of a base and h is the height
* Theorem 12.8 – volume of a cylinder – the volume V of a cylinder is V=Bh= π r2h, where B is the area of a base, h is the height, and r is the radius of a base
* Theorem 12.9 – volume of a pyramid – the volume V of a pyramid is V=⅓Bh, where B is the area of a base and h is the height
* Theorem 12.10 – volume of a cone – the volume V of a cone is V=⅓Bh= ⅓π r2h, where B is the area of a base, h is the height, and r is the radius of a base
* **Sphere** – the locus points in space that are a given distance from the center of the sphere
* **Radius of a sphere** – a segment from the center to a point on the sphere
* **Chord of a sphere** – a segment whose endpoints are on the sphere
* **Diameter** – a chord that contains the center
* **Great circle** – the intersection of a sphere and a plane that contains the center of the sphere
* **Hemisphere** – half of a sphere
* Theorem 12.11 – surface area of a sphere – the surface area S of a sphere with radius r is S=4πr2
* Theorem 12.12 – volume of a sphere – the volume V of a sphere is V=4/3π r3